First Fundamental Theorem Of Calculus

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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Question 1

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation, Limits and Continuity

Subtopics: Properties of Integrals, Fundamental Theorem of Calculus (First), Concavity, Tangents To Curves, Mean Value Theorem, Continuities and Discontinuities, Derivative Tables

Paper: Part B-Non-Calc / Series: 2002 / Difficulty: Hard / Question Number: 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

6. Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\, dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

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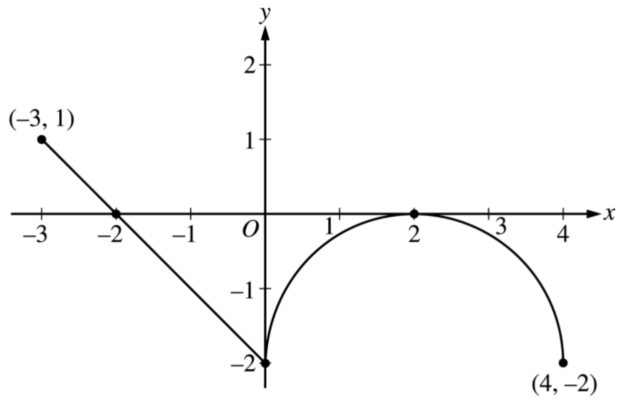
Question 2

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Derivative Graphs, Increasing/Decreasing , Points Of Inflection, Tangents To Curves, Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (First)

Paper: Part B-Non-Calc / Series: 2003 / Difficulty: Hard / Question Number: 4



Graph of f'

4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

Question 3

Qualification: AP Calculus AB

Areas: Differentiation, Integration, Applications of Integration, Applications of Differentiation

Subtopics: Rates of Change (Average), Riemann Sums – Trapezoidal Rule, Average Value of a Function, Fundamental Theorem of Calculus (First), Interpreting Meaning in Applied Contexts, Mean Value Theorem

Paper: Part A-Calc / Series: 2005 / Difficulty: Very Hard / Question Number: 3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

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Question 4

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Total Amount, Local or Relative Minima and Maxima, Fundamental Theorem of Calculus (First), Average Value of a Function

Paper: Part A-Calc / Series: 2009 / Difficulty: Easy / Question Number: 2

2. The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

(a) How many people are in the auditorium when the concert begins?

(b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

(c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

(d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

Question 5

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Rates of Change (Average), Fundamental Theorem of Calculus (First), Riemann Sums – Left

Paper: Part B-Non-Calc / Series: 2009 / Difficulty: Medium / Question Number: 5

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

(a) Estimate $f'(4)$. Show the work that leads to your answer.

(b) Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

(d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

Question 6

Qualification: AP Calculus AB

Areas: Differentiation, Applications of Integration

Subtopics: Modelling Situations, Total Amount, Implicit Differentiation, Interpreting Meaning in Applied Contexts, Fundamental Theorem of Calculus (First)

Paper: Part A-Calc / Series: 2009-Form-B / Difficulty: Easy / Question Number: 1

1. At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\, dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

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Question 7

Qualification: AP Calculus AB

Areas: Applications of Integration, Integration

Subtopics: Rates of Change (Average), Riemann Sums – Trapezoidal Rule, Interpreting Meaning in Applied Contexts, Fundamental Theorem of Calculus (First)

Paper: Part A-Calc / Series: 2011 / Difficulty: Easy / Question Number: 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

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Question 8

Qualification: AP Calculus AB

Areas: Applications of Integration, Integration, Differentiation

Subtopics: Rates of Change (Average), Interpreting Meaning in Applied Contexts, Fundamental Theorem of Calculus (First), Riemann Sums – Left

Paper: Part A-Calc / Series: 2012 / Difficulty: Medium / Question Number: 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

Question 9

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Mean Value Theorem, Local or Relative Minima and Maxima, Derivative Tables, Differentiation Technique – Chain Rule, Fundamental Theorem of Calculus (First)

Paper: Part B-Non-Calc / Series: 2014 / Difficulty: Somewhat Challenging / Question Number: 5

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

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Question 10

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Derivative Tables, Tangents To Curves, Differentiation Technique - Quotient Rule, Fundamental Theorem of Calculus (First), Differentiation Technique – Chain Rule

Paper: Part B-Non-Calc / Series: 2016 / Difficulty: Medium / Question Number: 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

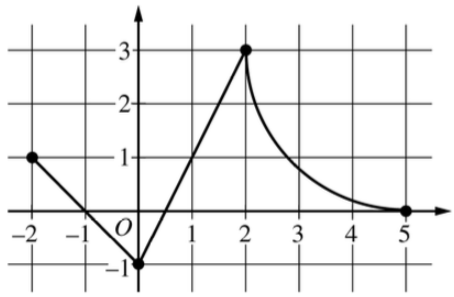
Question 11

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (First), Global or Absolute Minima and Maxima, Calculating Limits Algebraically, Integration Graphs

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Medium / Question Number: 3



Graph of f

3. The continuous function f is defined on the closed interval $-6 \leq x \leq 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f.

(a) If $\int_{-6}^{5} f(x)\,dx = 7$, find the value of $\int_{-6}^{-2} f(x)\,dx$. Show the work that leads to your answer.

(b) Evaluate $\int_{3}^{5} (2f'(x) + 4)\,dx$.

(c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\,dt$. Find the absolute maximum value of g on the interval $-2 \leq x \leq 5$. Justify your answer.

(d) Find $\lim_{x \to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x}$.

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Question 12

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Derivative Tables, Differentiation Technique – Chain Rule, Concavity, Differentiation Technique – Product Rule, Fundamental Theorem of Calculus (First), Increasing/Decreasing

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: Somewhat Challenging / Question Number: 5

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\,dt$. Find $m(2)$. Show the work that leads to your answer.

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.